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A POSTSCRIPT TO
"DYNAMIC PROBLEMS IN THE THEORY OF THE FIRM"

BY
HARVEY M. WAGNER

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TECHNICAL REPORT NO. 72
MARCH 23, 1959

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A POSTSCRIPT TO "DYNAMIC PROBLEMS IN THE THEORY OF THE FIRM"

by

Harvey M. Wagner
Stanford University

I. Introduction.

In a previous paper [2] we discussed the combined use of traditional economic constructs and dynamic programming for solutions to several inter-temporal problems in the theory of the firm (that produces a single commodity).¹ As a specific application of these techniques, we presented [3] a method for solving a dynamic version of the economic lot size model. The purpose of this postscript is (i) to demonstrate the algorithm suggested for the dynamic economic lot size model is also applicable to situations in which the cost curves (which may differ from period to period) have non-increasing marginal costs as a function of output², e.g., due to quantity discounts, and (ii) to describe a forward algorithm for solving optimal pricing and output problems in which the cost curves have non-decreasing marginal costs as a function of output.³

¹ The reader should recognize the parallel existing between "production" and "ordering inventory;" for ease of exposition, we frame our discussion in the language of "production."

² Figuratively "increasing marginal returns to scale."

³ Figuratively "decreasing marginal returns to scale."

Throughout we assume that each marginal revenue relation is a non-increasing function of the corresponding period sales, and initial and ending inventory are zero.⁴

We consider finding a sequence of output $q_o(t)$ and prices, which in turn determine sales $q_s(t)$, that maximizes total profits over the entirety of periods $t = 1, 2, \dots, T$. The period marginal revenue and cost functions are denoted by $MR[q_s(t)]$ and $MC[q_o(t)]$, and the unit cost of carrying an item of inventory from period t to period $t+1$ is denoted by i_t .

It will be helpful to recall the previous system of diagrams [2, p. 57], arraying each period's marginal revenue and cost curves, constructed such that the second period's curves are shifted downward by the amount i_1 , the third period's by the amount $i_1 + i_2, \dots$, and the T -th period's by the amount $i_1 + i_2 + \dots + i_{T-1}$. We permit the marginal functions to have horizontal segments (representing a constant unit cost or price over a range of values for output or sales), and adopt the convention of connecting two "adjacent" horizontal segments by a vertical segment.⁵ It should be noted that in the commonly considered case of a fixed amount demanded S_t for each period t (at some fixed price p), the marginal revenue curve is represented as a step function

⁴ In several cases only trivial alterations in the algorithms are needed to remove these assumptions.

⁵ In other words, if a marginal curve is in part a step function, the "rise" in the step is drawn as well as the "flat."

$$MR[q_s(t)] = \begin{cases} p & \text{if } q_s(t) \leq S_t \\ 0 & \text{otherwise.} \end{cases}$$

In the case of a capacity constraint on period production, the marginal cost curve is depicted as rising vertically at this point.

II. The Case of Non-Increasing Marginal Costs.

In [2,3] we discussed the model in which the marginal cost curve is horizontal⁶ and identical in all periods, and non-negative setup costs are incurred if production takes place. The suggested forward algorithm is briefly described as:

At period t^* (starting with $t^* = 1$) compute the profits from producing in period t^{**} ($t^{**} = 1, 2, \dots, t^*$), and filling all $q_s(t)$,⁷ $t = t^{**}, t^{**} + 1, \dots, t^*$, by production in period t^{**} . Add to this figure the profits of using an optimal policy in periods $t = 1, 2, \dots, t^{**}-1$ considered by themselves.⁸ Select producing in the t^{**} that offers the maximum total profits; this yields an optimal policy for the first t^* periods. Continue until $t^* = T$.

That this algorithm leads to an optimal solution is based on the fundamental proposition [3, p. 91]: There exists an optimal program such that at any period t one need not both produce and bring in inventory.

⁶ Consequently capacity constraints are ignored.

⁷ See [2, p. 61] for the method of determining $q_s(t)$. In the commonly considered case of a fixed amount demanded for each period, $q_s(t)$ are of course pre-determined.

⁸ Since the algorithm starts at $t^* = 1$, these profit figures have been previously computed.

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We assert that the same algorithm leads to an optimal solution if the marginal cost curves are non-increasing and not necessarily identical in all periods; as before, non-negative setup costs may be included. The reason underlying the assertion is that the fundamental proposition continues to hold. We sketch a proof:

Suppose a trial solution indicates in period t^* that both inventory is brought in and production takes place. Specifically, let $\{P\}$ denote the set of periods supplying inventory to period t^* . Consider the current value of marginal cost for each $t \in \{P\}$, and add to this number the carrying costs charged for bringing an additional unit of production from t to t^* . Let MC' be the minimum value of these sums and t' the associated period. If MC' is less than the MC for the last unit produced in period t , we may revise the trial solution so as to increase production in period t' and eliminate it in period t . If MC' is not less than the MC of the last unit produced in period t , we may revise the trial solution so as to increase production in period t equal to the amount of incoming inventory and eliminate the corresponding production in $\{P\}$. Such alterations do not increase the total cost over that of the trial solution because of our hypothesis of decreasing marginal cost functions.

We should recognize that our present assumption about marginal costs is too weak to prove the Planning Horizon Theorem [3], which in part states that if it is optimal to produce when periods 1 through t^* are considered by themselves, then it is sufficient to consider programs for all T periods such that production takes place in period t^* . In Table 1 we present an

example in which the proposition is violated; the optimal two-period program is to produce in period 2, whereas the optimal three-period program is to produce only in period 1. Consequently, in the general case of non-increasing marginal costs, the addition of a new period $t^* + 1$ may cause a major revision in an optimal plan for periods 1 through t^* .

Table 1

	Period 1	Period 2	Period 3
Setup Cost	12	2	5
Unit Carrying Charge	0	0	0
Unit Cost	8	9	10
Selling Price	15	15	15
Amount Demanded	0	3	20

III. The Case of Non-Decreasing Marginal Costs.

In [2] we described a recursive method for solving the class of problems in which marginal cost is non-decreasing. Essentially the technique involved finding a first trial solution for all T periods; if the plan proved to be infeasible⁹, finding a second trial solution to a smaller number of periods; and so forth, until finding a feasible solution for periods $t = 1, 2, \dots, t^* \leq T$. If $t^* < T$, the method is repeated for periods $t = t^* + 1, \dots, T$.

⁹ I.e., at some period, cumulative sales exceeded cumulative production.

Here we note that the ingenious method of S. Johnson [1] extends immediately to our problem to provide an alternative mode of attack. Specifically, it is possible to construct an optimal sales and production pattern utilizing an algorithm starting at $t = 1$, and successively adding the data for each following period.

We define a "cascade function," to be superimposed on the system of diagrams arraying each period's marginal curves, as a step function having a single horizontal step for each period and such that the heights of the steps are non-increasing over time. Figure 1 shows an example of a cascade function for a three-period model.

The algorithm for constructing an optimal solution proceeds as follows:

Step 1. In period 1 define the height of the step as the value of the marginal cost curve at the output for which period 1 marginal cost and revenue curves intersect. If the intersection is at a vertical segment of the MC curve, then the height is defined as the value of MC on the immediate left of the intersection point.¹⁰

The provisional values for $q_s(1)$ and $q_o(1)$ are determined as the output at the intersection of the MR and MC curves. If the intersection is non-unique, then the largest saleable output on the locus of the intersection is selected.

Step 2. The height of the cascade step for period 2 is provisionally set at the height of the step for period 1. The trial value for $q_o(2)$ is the smallest output such that $MC[q_o(2)] =$ the height of the trial

¹⁰ It may happen that at $q_o(1) = q_s(1) = 0$, the MC curve lies above the MR curve, in which case it is never profitable to sell any item in period 1; the height of the cascade step is then defined as the value of the MC curve on the immediate right of $q_o(1) = 0$.

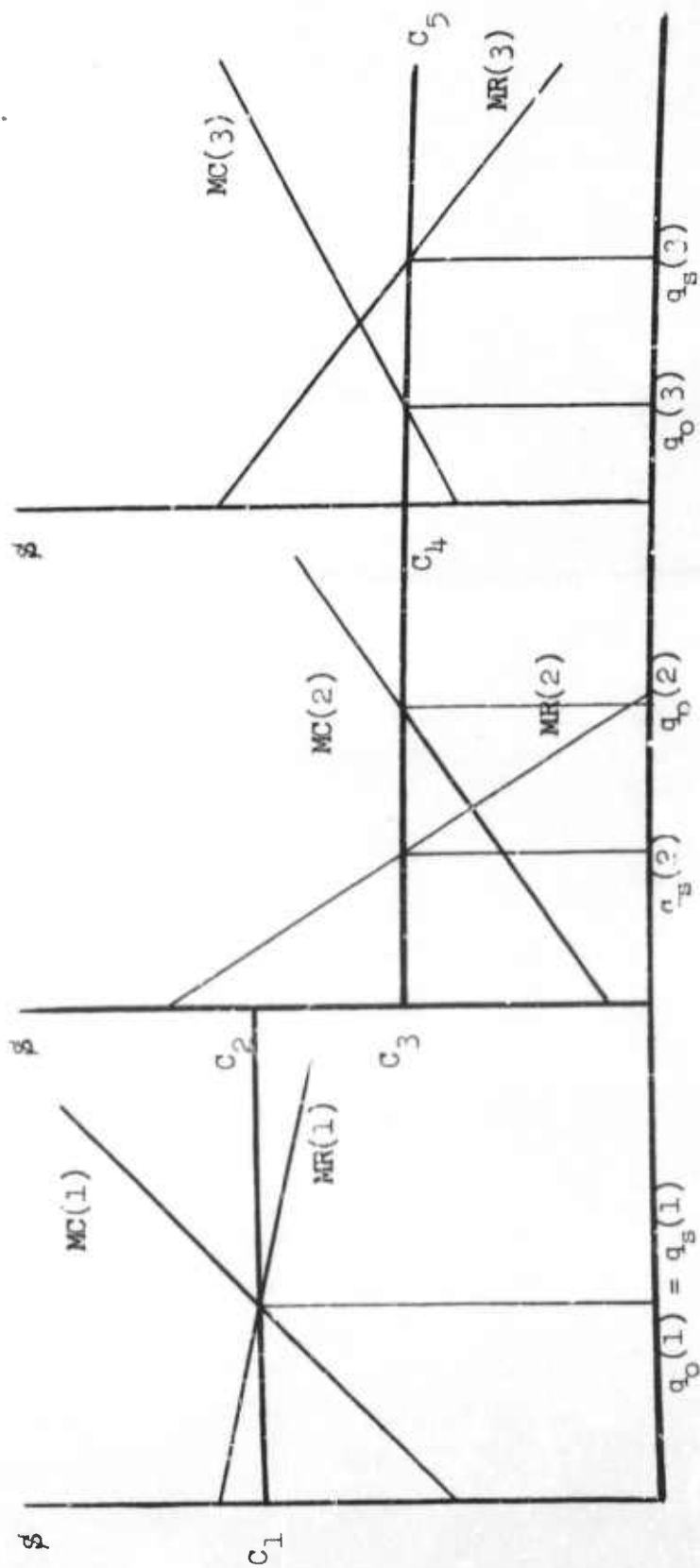


Fig. 1

step, and for $q_s(2)$ is the largest sales figure such that $MR[q_s(2)] =$ the height of the trial step.

Case (i). If $q_o(2) = q_s(2)$, no alteration is made in the provisional values.

Case (ii). If $q_o(2) > q_s(2)$, then the height of the step for period 2 must be lowered until equality obtains between the amount sold and produced in period 2. In lowering the step, if a horizontal section of the MR curve is reached, the sales figure should be increased as much as possible toward meeting the equality condition. If a horizontal section of the MC curve is reached, output should be curtailed as much as possible toward meeting the equality condition. We adopt the rule that if horizontal sections in the MR and MC functions are met simultaneously, the increase in sales takes precedence over the decrease in output.

Case (iii). If $q_o(2) < q_s(2)$, then in this instance cumulative output over the two periods is not sufficient to meet cumulative demand. If possible increase output in period 2 at the same marginal cost, with a view toward meeting the condition that cumulative output equals cumulative sales. Should the condition remain unsatisfied, if possible increase output in period 1 at the same marginal cost. Should the condition then remain unsatisfied, if possible decrease sales in period 2 at a value of marginal revenue equal to the provisional height of the step. Should the condition continue to be unsatisfied, if possible decrease sales in period 1 at a value of marginal revenue equal to the provisional height of the step. Should the condition still remain unsatisfied, then raise the height of the step, maintaining the cascade property by raising the

height of the step for period 1 and altering, mutatis mutandis, $q_0(t)$, $q_s(t)$, $t = 1, 2$, until equality obtains between cumulative sales and production.

In raising the height of the step, if horizontal segments in the MR and MC curves are encountered, an increase in output takes precedence (starting at the latest period being considered) over a decrease in sales.

Step t^* . The height of the cascade step for period t^* is provisionally set at the height of the step for period $t^* - 1$. As in Step 2, trial values for $q_0(t^*)$ and $q_s(t^*)$ are determined.

Case (i). If $q_0(t^*) = q_s(t^*)$, then no alteration is made in the provisional values.

Case (ii). If $q_0(t^*) > q_s(t^*)$, then the alteration given in Step 2, Case (ii), is applied here.

Case (iii). If $q_0(t^*) < q_s(t^*)$, then if possible increase output in period t^* at the same marginal cost, with a view toward meeting the condition that cumulative output ($t = 1, 2, \dots, t^*$) equals cumulative sales. Should the condition remain unsatisfied, if possible increase output in periods $t^* - 1$, $t^* - 2, \dots$, at the same marginal cost.¹¹ Should the condition continue to be unsatisfied, if possible decrease sales in period $t^* - 1$, $t^* - 2, \dots$, at a value of marginal revenue equal to the provisional height of the step. Finally, should the condition still remain unsatisfied, then raise the height of the step, maintaining the cascade property by raising the height of the step for period t^*-1 and any

¹¹ I.e., for all immediately preceding periods having the cascade function at the same height as the current provisional value.

previous periods as they become eligible, and altering, mutatis mutandis, $q_0(t)$, $q_s(t)$, $t = 1, 2, \dots, t^*$, until equality obtains between cumulative sales and production. As in Step 2, Case (iii), increasing output takes precedence over decreasing sales whenever such simultaneous alternatives present themselves.

In summary, the algorithm calls for the successive adding of new period demand and supply relationships, and effecting revisions in a fashion such that, at any given period, planned sales never increase and planned production never decreases. The construction consequently maintains feasibility, for the adding of a new period either leaves the previous periods' plans unchanged and the new period is considered by itself, or causes a revision of the previous plans such that cumulative demand (at any period) decreases and cumulative production increases. The cascade function ensures the marginal equalities necessary for a constrained local optimal feasible solution, and the assumptions that MR is a non-increasing function and MC a non-decreasing function guarantee that any discovered local optimum is also a global optimum.

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